MATH 8 UNIT 2

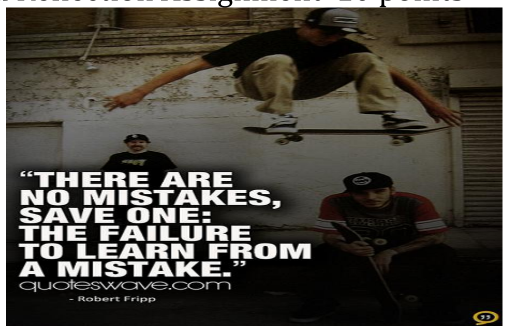
Introduction to Trigonometry, Graphs of Sine and Cosine, Trig Equations i

NAME: ____

	Unit 2 Homework Checklist	
	Test 1 Rework	*10*
NOTES	*completed and in order given including completd worksheets:*	*10*
	WS pg 9: Soving equations with sine/cose part 1	*5*
	WS pg19-21 Graphing sine/cosine part 1	*5*
B2i	1-3, 5, 7, 9, 15-25 odd, 29-32	*5*
10.2i	*1-38*	*5*
10.3ii	*6-17*	*5*
10.4i	1-24, 39-69 odd	*5*
10.4ii	25-37 odd and page 839 39 and 40 (all trig values)	5
11.1	1-11, 26, 29, 35, 37, 39, 49, 59, 67, 71, 74	5
	Sample Test	10
		70

Unit 2– Introduction to Trigonometry, Graphs of Sine and Cosine, Trig Equations i.

Test 1 Test Corrections and Reflection Assignment 10 points



Rework your exam as described below. This is a good study skill that you should do with <u>every</u> exam and it provides a valuable tool when studying for the final.

Rework Instructions:
DO NOT ERASE ANYTHING ON YOUR TEST. You can't learn from your mistakes if you just ignore or erase them.
Rework any problems you missed. If you don't know how to do it and don't understand the solutions I have posted, come see me or go to tutoring. Look at your work and try to figure out WHY it is wrong and perhaps figure out what you were thinking. Write yourself notes to explain the correct thought process. Either use a different color and write next to your work, write on post-it notes and attach them right next to your work, or write on attached pages. Put the reworked test in your notebook right after this page. Then answer the following questions. Do not give answers that you think I want to hear, ("I will study more") think deeply and give specific, honest answers.

(1) What, specifically, did you do to prepare for this test?

(2) Did you think you were well prepared going in to this test? Do you think so now, having seen your mistakes?

(3) Looking at your mistakes, what were your weaknesses on this material?

(4) If your score was below 70, what, *specifically*, will you do differently this time?

Unit 2– Introduction to Trigonometry, Graphs of Sine and Cosine, Trig Equations i.

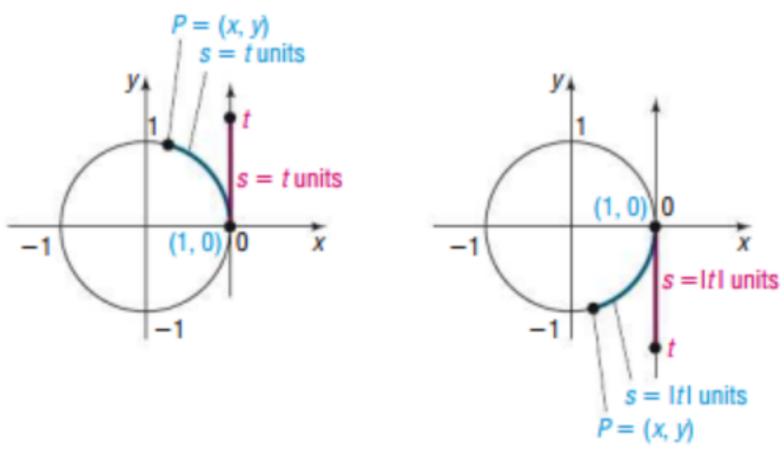
10.2i Intro to Sine and Cosine Circular Functions

Students should review functions as needed: Functional notation, domain, range, inverses, graphs etc.

We will look at three definitions of the trigonometric functions, each useful in different situations.

The Unit Circle Definition of Sine and Cosine Function.
Given any real number t, we define the functions sin(t) ("______________________") and cos(t) ("______________________") by the following process.

Consider the real number line corresponding values of t aligned next to the unit circle as shown. If this number line were wrapped around the unit circle, then every number t would correspond to a point P(x,y) on the unit circle $x^2+y^2=1$ found by using t as the arc length. We will discuss the physical idea behind this definition when we graph these functions.

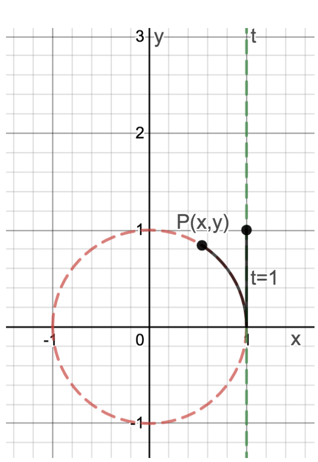


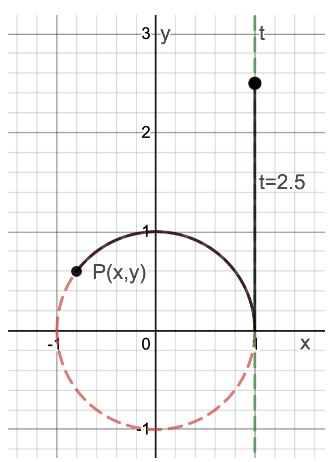
$\cos(t) =$ ____________

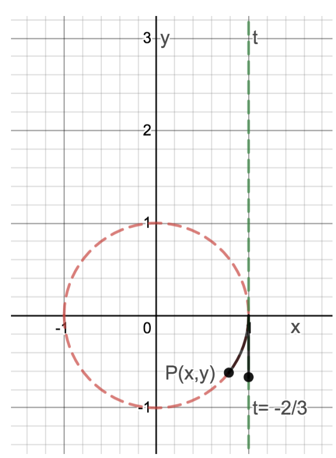
We define cosine of t to be the x value of that point and sine of t to be the y value.

$\sin(t) =$ ____________

Examples: Approximating cosine and sine values.



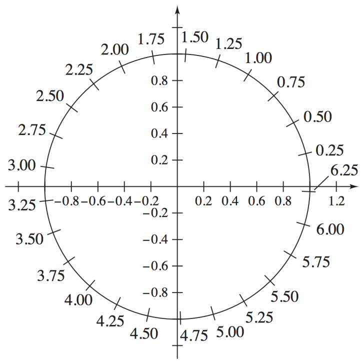




<u>Examples: Approximating cosine and sine values using “unit circle wrap”.</u>

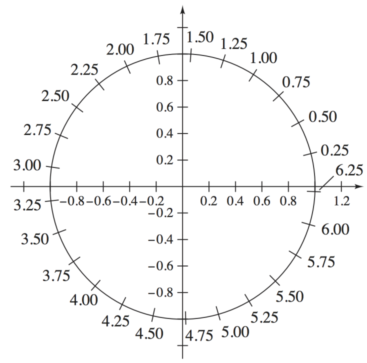
$\cos(2.75) \approx$ ____________

$\sin(2.75) \approx$ _____________



$\cos(\quad) \approx$ ____________

$\sin(\quad) \approx$ _____________



<u>Examples: Approximating cosine and sine values using calculator</u> (How does IT do it?)

$\cos(2.75) \approx$ ____________

$\sin(2.75) \approx$ _____________

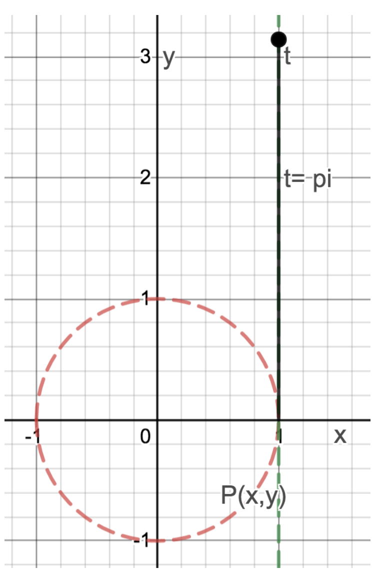
Important Note: when input is a real number, mode is radian as explained later. Notice that at this point, the trig functions have NOTHING to do with angles. The input and output are numbers.

In all the above examples, we can only APPROXIMATE the sine and cosine function values. Can we ever compute them EXACTLY?

$\cos(\pi) =$ ____________

(EXACTLY)

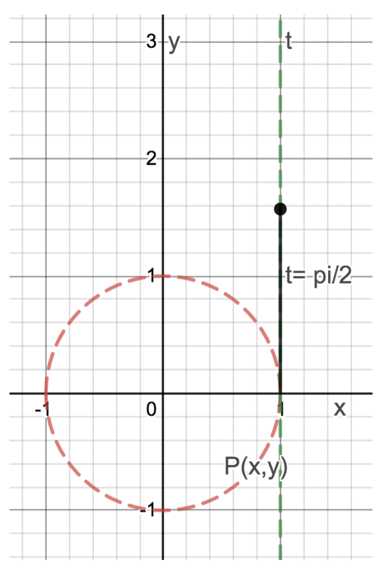
$\sin(\pi) =$ _____________



$\cos(\pi/2) =$ ____________

(EXACTLY)

$\sin(\pi/2) =$ _____________



So if for a given input t, we know the exact coordinates of P, we can find the sine and cosine.

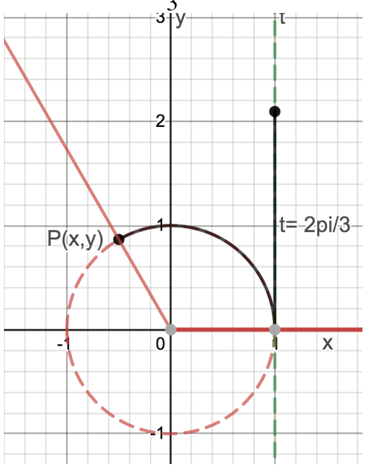
<u>Finding EXACT trigonometric values for special cases.</u>

You may already see this relationship but in our definition of a radian we found $\frac{s}{r}=\theta$, so $s=r\theta$. Thus in the unit circle ______________________

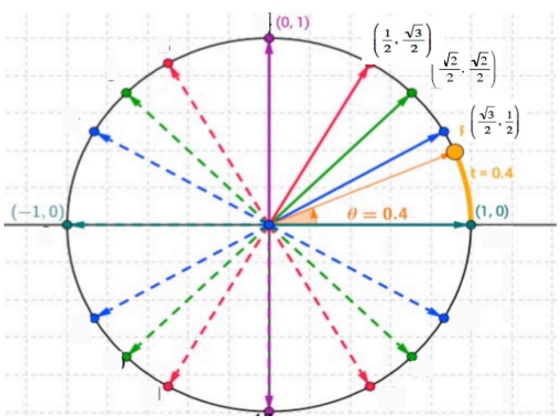
That is, the point on the unit circle corresponding to a input of t is the same point we obtain using an angle of t radians which we have already been finding.

So even though we are not officially using angles when discussing trig values for real number inputs t, we can use our *knowledge* of angles to locate the point on the unit circle corresponding to t.

Example: $t=\frac{2\pi}{3}$



Putting this together with our knowledge from the last unit, we are just finding the x or y value of the point where the terminal side of the angle corresponding to the arc length t intersects the unit circle.



Examples:

$\sin\left(\frac{\pi}{6}\right)=$________________ $\cos\left(\frac{3\pi}{2}\right)=$________________ $\sin\left(\frac{-\pi}{4}\right)=$______________

$\cos\left(\frac{5\pi}{4}\right)=$________________ $\sin\left(\frac{4\pi}{3}\right)=$________________ $\cos\left(\frac{13\pi}{6}\right)=$_________________

More practice: https://www.thatquiz.org/tq-q/?-j43-l1-p2kc0

Function Properties of Sine and Cosine

Keep in mind that $\sin(t)$ and $\cos(t)$ are functions and in using this notation, we are using functional notation and t is called the input or the argument.

Cautionary Examples: $\sin(3t)$ $\sin(a+b)$ $\sqrt{\ }$

In our previous studies of important functions, we often consider characteristics like domain, range, graph, inverse function, solving equations and applications. We will do the same for these functions.

What would be the domain and the range of $\sin(t)$ and $\cos(t)$?

> **Definition 10.3. Periodic Functions:** A function *f* is said to be **periodic** if there is a real number *c* so that *f*(*t* + *c*) = *f*(*t*) for all real numbers *t* in the domain of *f*. The smallest positive number *p* for which *f*(*t* + *p*) = *f*(*t*) for all real numbers *t* in the domain of *f*, if it exists, is called the **period** of *f*.

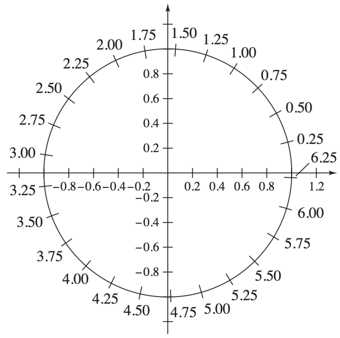
Notice that $\sin(t)$ and $\cos(t)$ are ______________________ with period ____________

A function is said to be **even** if $f(-t) = f(t)$. _________________ is an even function. We can use this fact as another way to find value for a negative number input.

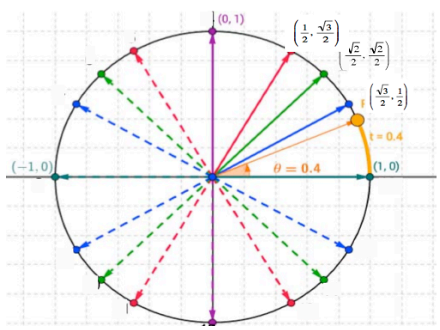
A function is said to be **odd** if $f(-t) = -f(t)$. _________________ is an odd function. We can use this fact as another way to find value for a negative number input.

10.2ii Introduction to Solving Trigonometric Equations
(Going backwards from finding trig. Values)

Using "Unit Circle Wrap" idea;



Special Number Inputs



Solve: $\sin(t)=\frac{\sqrt{2}}{2}$

This is saying, find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has Y value of $\frac{\sqrt{2}}{2}$

Why Y value? ____________________________

How many terminal sides are there corresponding to this________________________

How many values of t? (or think in angles) ____________________________

How do we express infinitely many answers?____________________________

Sometimes we are asked to solve for t on a restricted domain:

Solve: $\sin(t)=\frac{\sqrt{2}}{2}$ for $0<t<\frac{\pi}{2}$ __________________________

Solve: $\sin(t)=\frac{\sqrt{2}}{2}$ for $0<t<2\pi$ __________________________

Solve: $\sin(t)=\frac{\sqrt{2}}{2}$ for $-2\pi<t<0$ __________________________

Solve: $\sin(t)=\frac{\sqrt{2}}{2}$ for $0<t<4\pi$ __________________________

Examples: While you are learning the process, I highly encourage you to draw the unit circle and find the location of the terminal sides corresponding to the solution.

Solve: $\cos(t) = \frac{-1}{2}$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has______________value of $\frac{-1}{2}$

Solutions: __________________________

Solve: $\cos(t) = \frac{-1}{2}$ for $0 < t < \pi$ __________________________

Solve: $\cos(t) = 1$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has______________value of 1

Solutions: __________________________

Solve: $\cos(t) = 1$ for $0 \le t < 2\pi$ __________________________

Solve: $\sin(t) = 0$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has______________value of 0

Solutions: __________________________

Solve: $\sin(t) = 0$ for $0 < t < \pi$ __________________________

Solve: $\sin(t) = -\frac{\sqrt{2}}{2}$

This is asking us to find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has______________value of $-\frac{\sqrt{2}}{2}$

Solutions: __________________________

Solve: $\sin(t) = -\frac{\sqrt{2}}{2}$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$ __________________________

Name:__________________

Worksheet: Solving equations with sine and cosine on restricted domain.

(1) Solve: $\sin(t) = 1$ (if no restrictions are given, list all solutions) ________________________

Solve: $\sin(t) = 1$ for $0 < t < 2\pi$ ________________________

Solve: $\sin(t) = 1$ for $0 < t < 6\pi$ ________________________

Solve: $\sin(t) = 1$ for $-2\pi < t < 0$ ________________________

Solve: $\sin(t) = 1$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$ ________________________

(2) Solve: $\cos(t) = 1/2$ ________________________

Solve: $\cos(t) = 1/2$ for $0 < t < 2\pi$ ________________________

Solve: $\cos(t) = 1/2$ for $-2\pi < t < 0$ ________________________

Solve: $\cos(t) = 1/2$ for $0 < t < \pi$ ________________________

(3) Solve: $\sin(t) = -\sqrt{3}/2$ ________________________

Solve: $\sin(t) = -\sqrt{3}/2$ for $0 < t < \pi$ ________________________

Solve: $\sin(t) = -\sqrt{3}/2$ for $0 < t < 2\pi$ ________________________

Solve: $\sin(t) = -\sqrt{3}/2$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$ ________________________

(4) Mixed

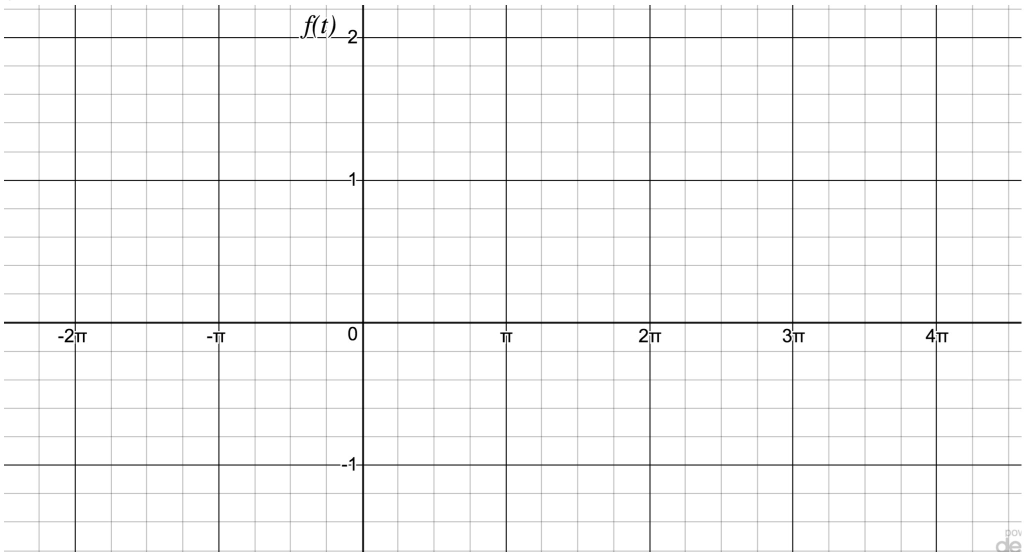
Solve: $\cos(t) = -\sqrt{3}/2$ for $0 < t < 2\pi$ ________________________

Solve: $\cos(t) = 0$ ________________________

Solve: $\sin(t) = \frac{\sqrt{2}}{2}$ for $0 \le t < 4\pi$ ________________________

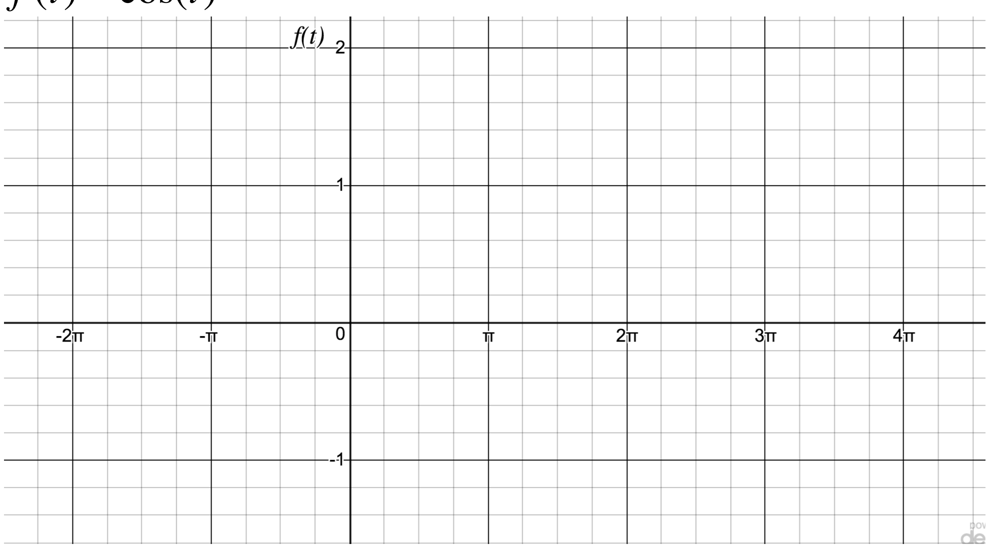
10.3i Graphing the Sine and Cosine Function part i

$f(t) = \sin(t)$



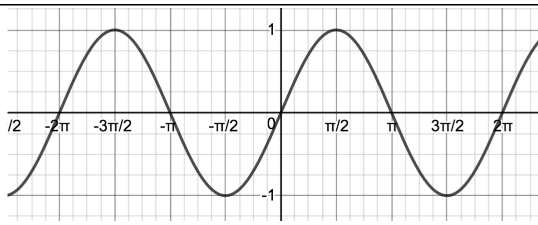
Note choice of scale on t axis.

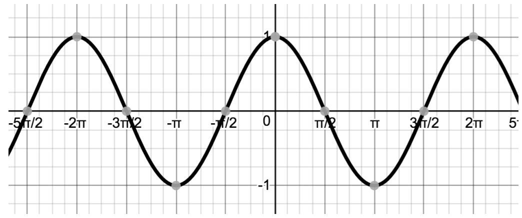
$f(t) = \cos(t)$



Note: On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points on graph.

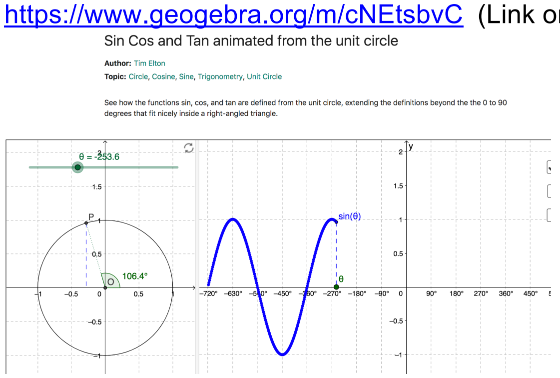
Discuss how domain, range, period, even/odd, can be seen on graph



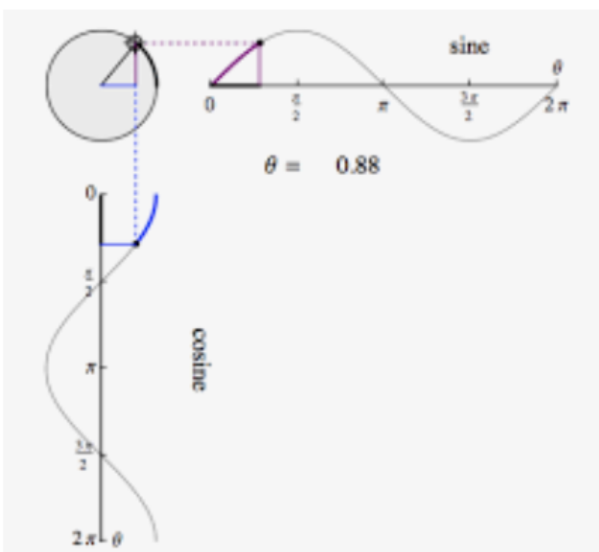


_______________________ _______________________

This type of "sinusoidal wave" can be used to measure many physical phenomena.
Annimation: See https://www.geogebra.org/m/cNEtsbvC (Link on Math 8 Page)



Also, consider the following graphic:



Both these graphs are ________________________ with period ___________________ and have key

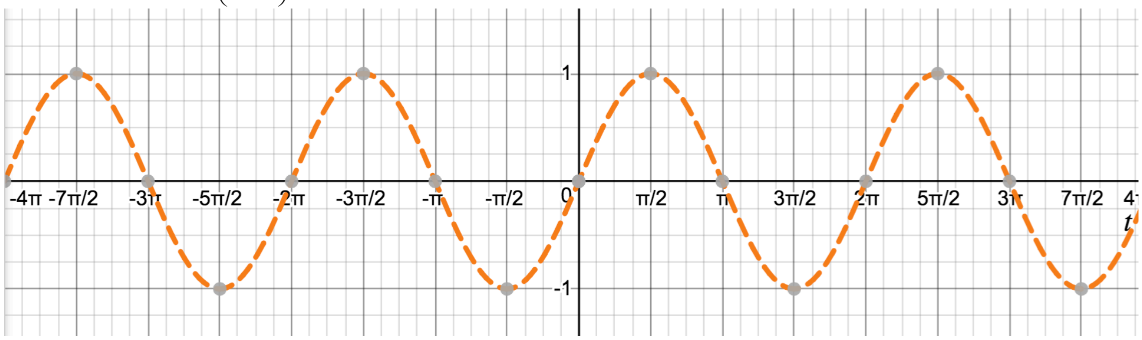
points occuring every quadrantal angle or every _______________________

<u>Transformations of the sine and cosine graphs.</u>

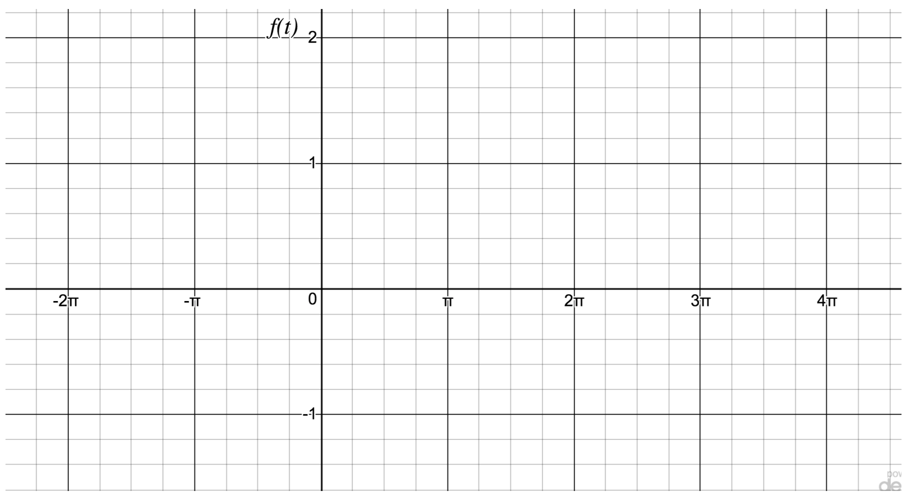
These two graphs can be used as basic graphs together with transformations (review 5.4 as needed).

Shift f(x+c), f(x-c)

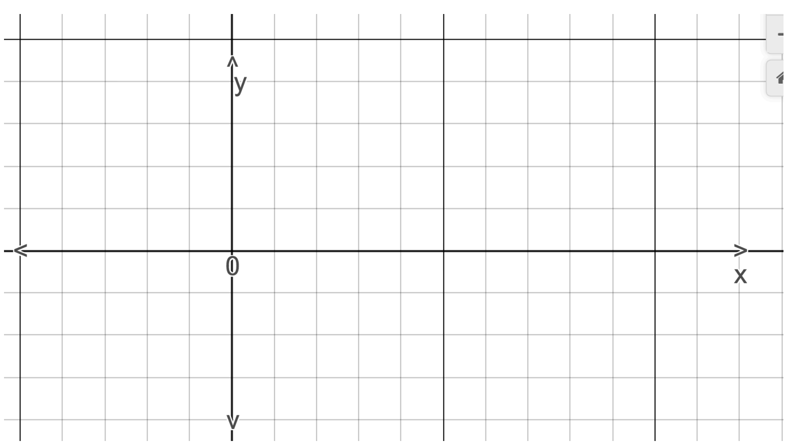
Ex. Graph $y = \sin\left(t - \frac{\pi}{4}\right)$, ______________________________



Ideally, eventually, rather than graph the original and then transform it, you would be able picture this transformation in you head to get a starting point, and then use the known pattern to generate the rest.

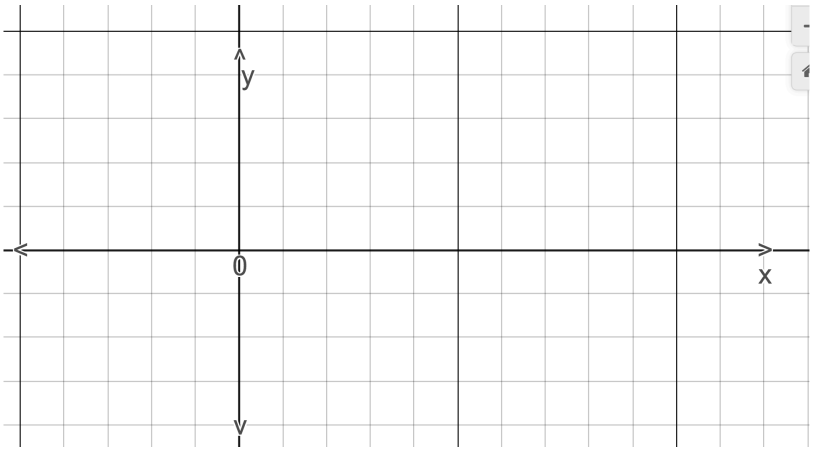


Graph $y = \cos(t + \pi)$ ______________________



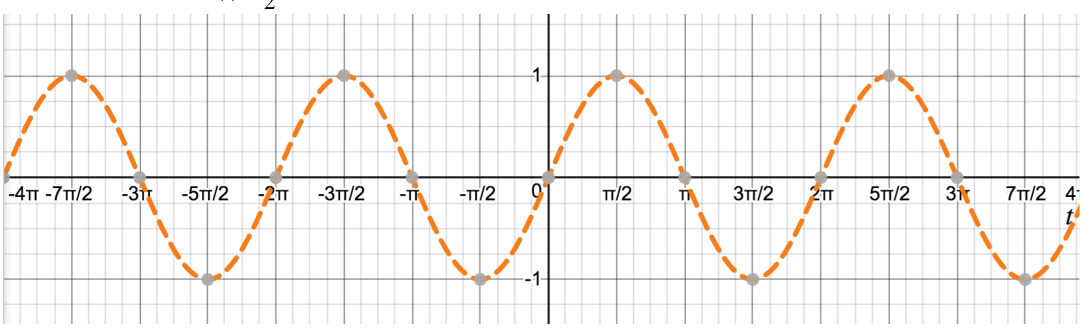
When graphing a sine or cosine graph, a choice of scale showing multiples of is usually a good choice, but in some cases, a better choice can be made.

Graph $y = \cos\left(t + \frac{\pi}{3}\right)$ ________________________________

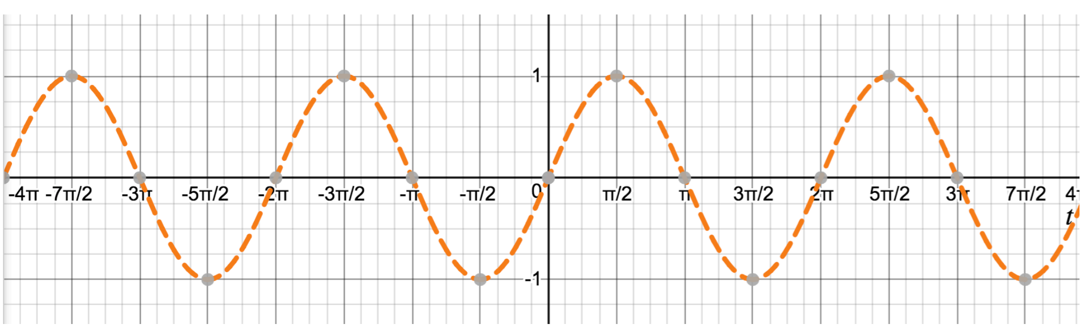


Vertical Shift ________________________________,

Ex. Graph $y = \sin(t) - \frac{1}{2}$ ________________, $y = \sin(t) + 1$ ____________

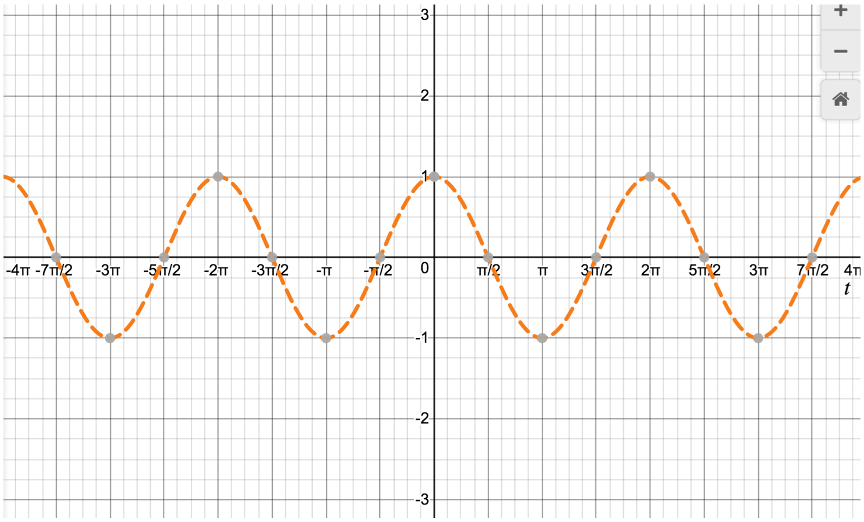


How would we graph $y = \sin\left(t - \frac{\pi}{8}\right) + \frac{1}{2}$? ________________________________

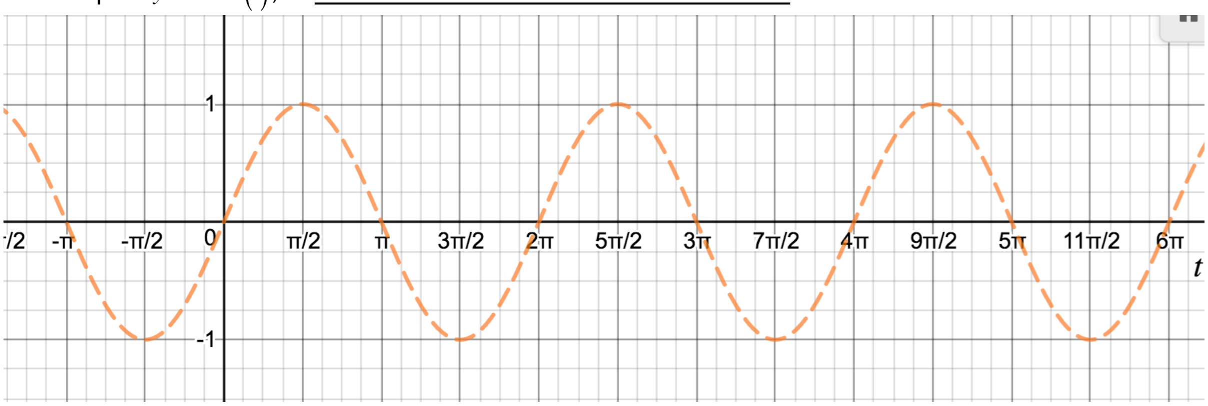


Vertical________________________________y=c f(x)__

Ex. Graph $y = 3\cos(t)$, $y = \frac{1}{2}\cos(t)$



Ex. Graph $y = -2\sin(t)$, ________________________________



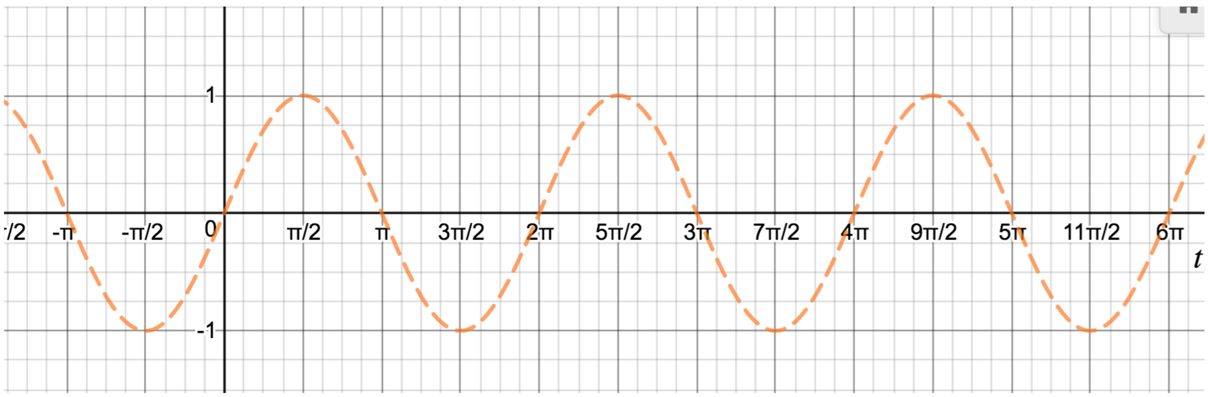
In general, for graphs of the form:

$$y = A\cos(t), \quad y = A\sin(t),$$

____

Horizontal stretch or compression ______________________________________

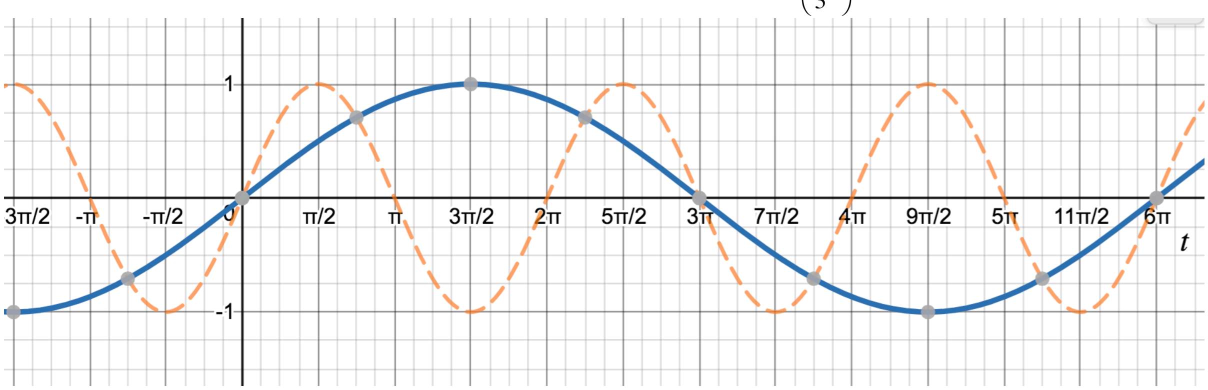
Graph $y = \sin(2t)$ ____



Initially, we might graph this by using our knowledge of horizontal compression or we night simply plot points (note: plotting points is inefficient and should be our last resource.)

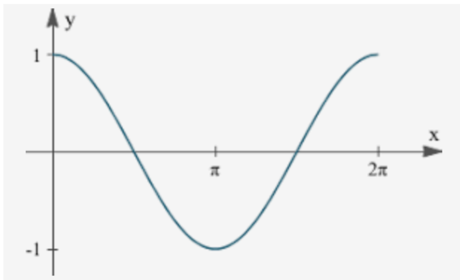
Period?_________________________

Thus, the above graph is a horizontal compression whereas $f(t) = \sin\left(\frac{1}{3}t\right)$ is a horizontal stretch.



Period ________________?

Side note: At this point, as a convention, we switch our input from t to x but keep in mind, this x is not the same as the x value of the point on the unit circle.
For example: $y = \cos(x)$ Explanation



In general, for graphs of the previous 2 examples involving horizontal strecth/compression:

$$f(x) = \cos(\omega x), \ f(x) = \sin(\omega x)$$

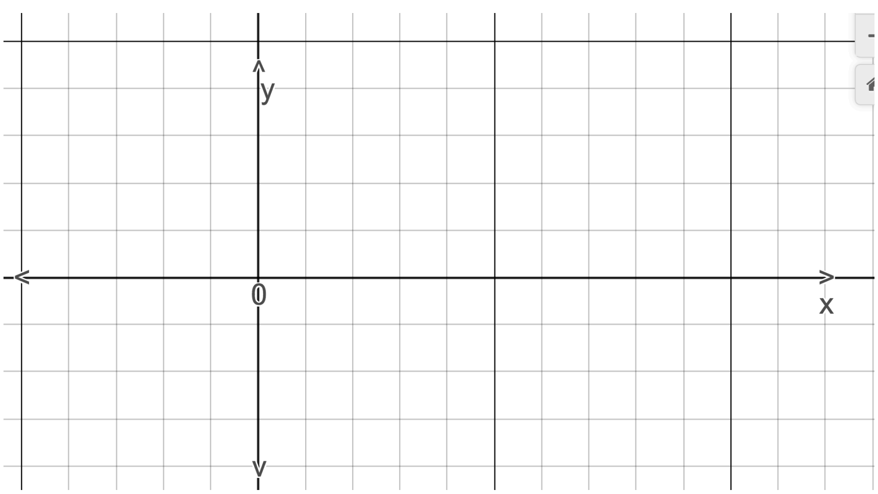
ω has the effect of changing the ___________ to_______________________ (new period)

Note: ω is the Greek ________________

For this type of graph, rather than sketch the original graph and then stretch/compress it, we plan ahead and find the period. Then we break this period into fourths since the key points (lo-zero-hi-zero) occur every one-fourth of the period, and choose our x axis scale accordingly.

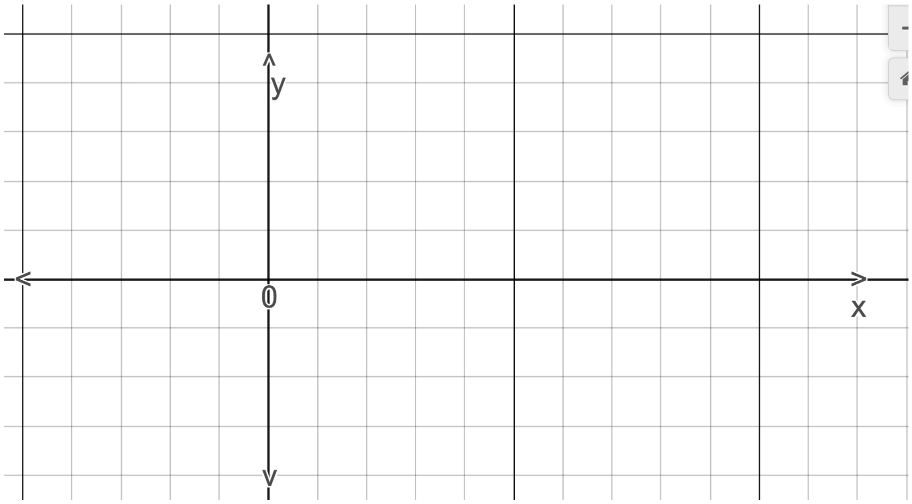
Reminder: On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points on graph

Ex: $f(x) =$



What is the period of this graph?

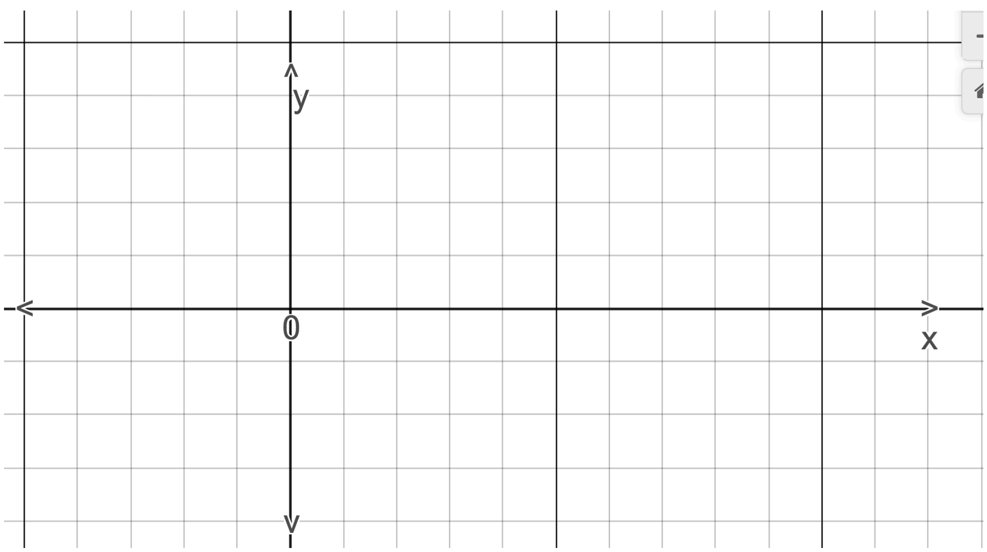
Ex: Graph at least one period of $f(x) =$.



Combining vertical and horizontal stretch/compress.

$$f(x) = A\cos(\omega x),\ f(x) = A\sin(\omega x)$$

Ex:



<u>Using a graph to visualize solutions to a trig equation.</u>

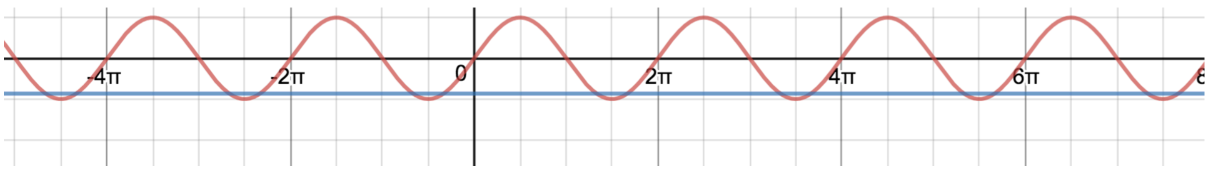
Recall from page 9,

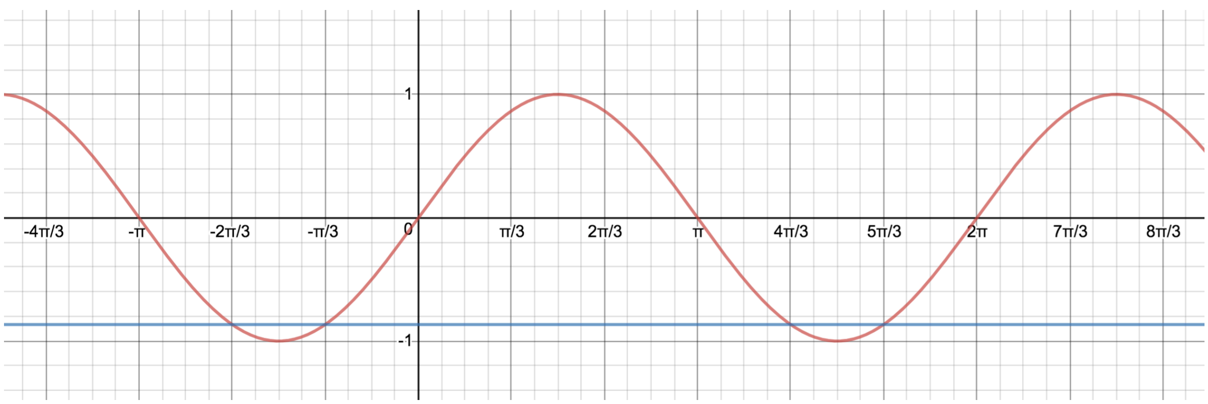
Solve: $\sin(t) = -\sqrt{3}/2$ ________________________

Solve: $\sin(t) = -\sqrt{3}/2$ for $0 < t < \pi$ ________________________

Solve: $\sin(t) = -\sqrt{3}/2$ for $0 < t < 2\pi$ ________________________

Solve: $\sin(t) = -\sqrt{3}/2$ for $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$ ________________________





Name: ______________________

Worksheet: Graphs Sine and Cosine part one (10.3i)

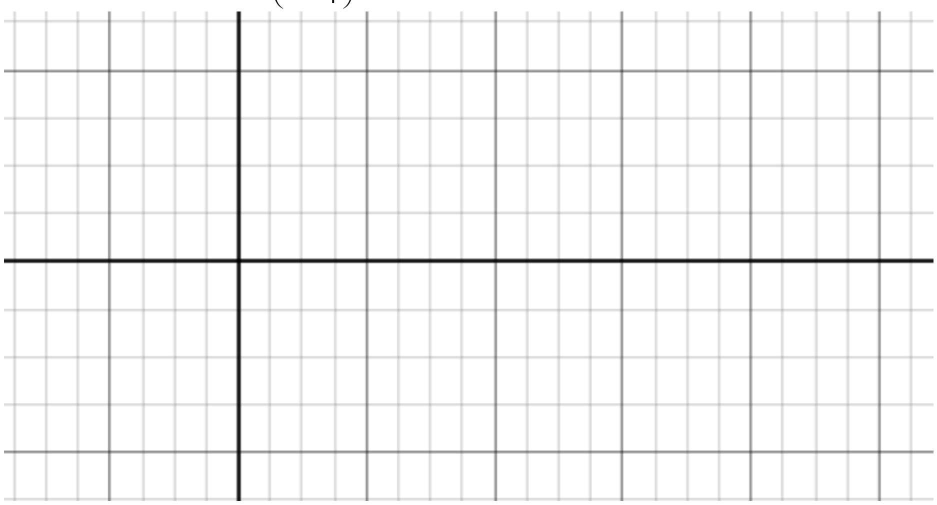
$$f(x) = A\cos(\omega x)\ ,\ \ f(x) = A\sin(\omega x)$$

What effect does A have on the graph?______________________________

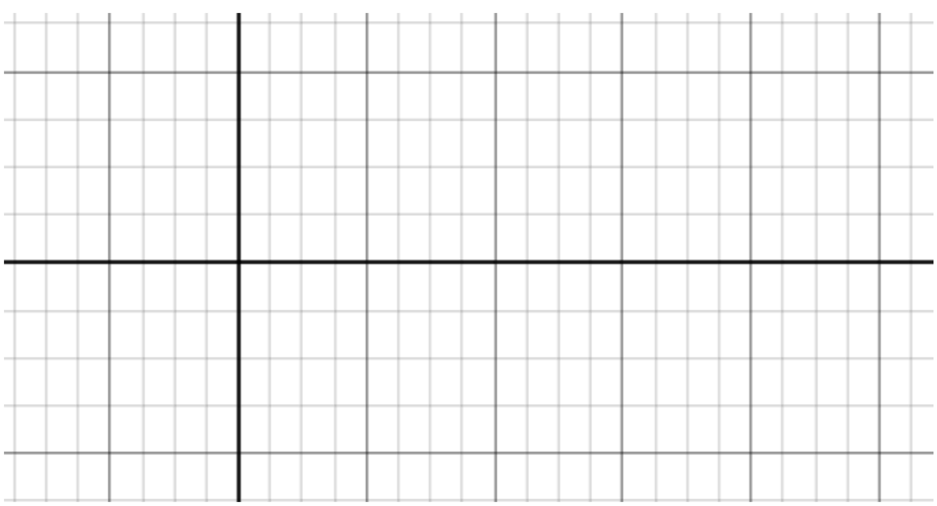
What effect does ω have on the graph?______________________________

Sketch at least one period of each of the following graphs. On all Trigonometric graphs, it is expected that you show scale clearly and label coordinates of high points and low points <u>on</u> graph

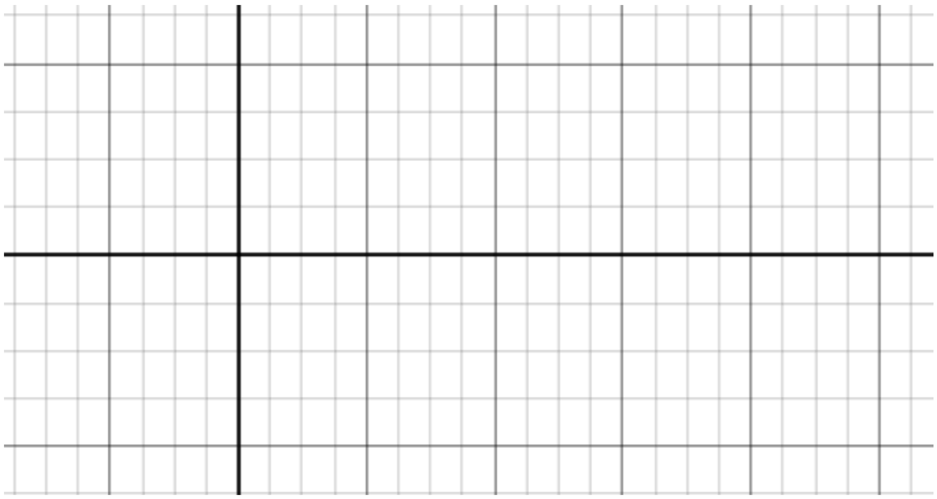
(1) Graph $f(x) = 4\sin\left(x - \frac{\pi}{4}\right)$



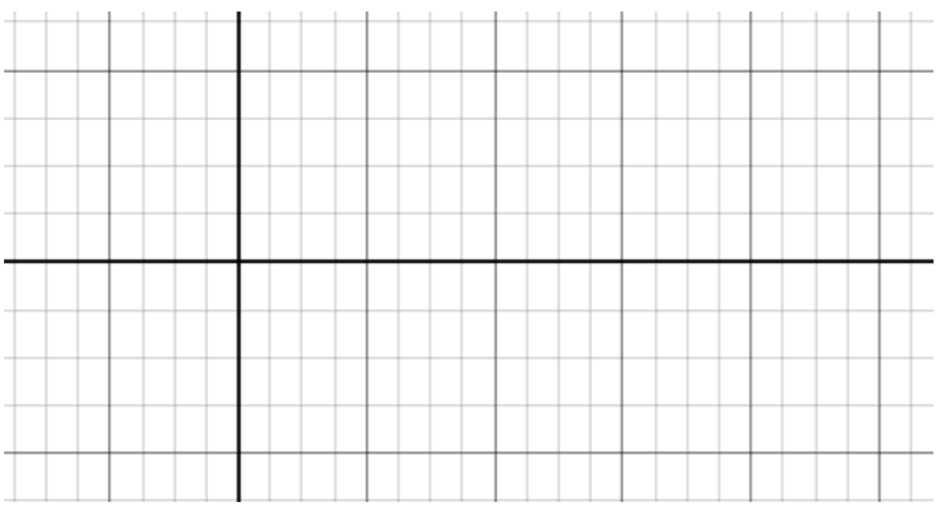
(2) Graph $f(x) = 3\cos\left(x - \frac{\pi}{3}\right)$



(3) Graph $f(x) = 2\sin\left(\frac{2\pi}{3}x\right)$



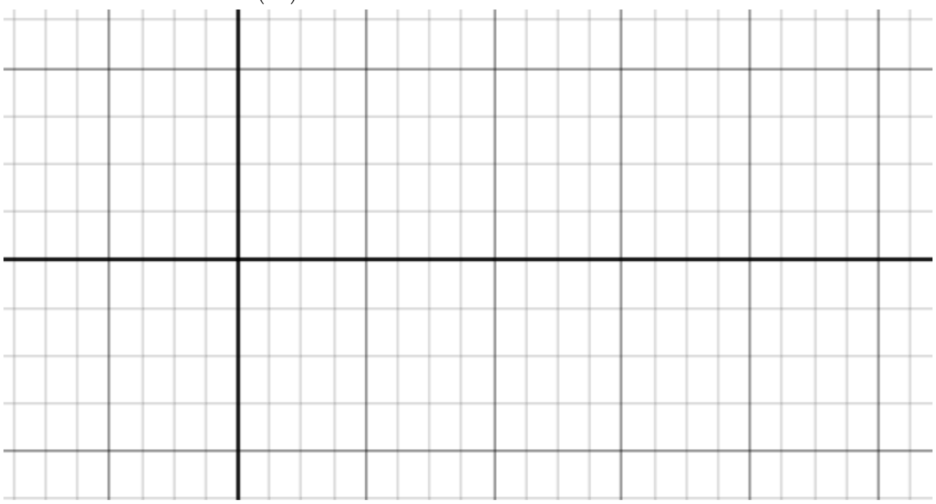
(4) Graph $f(x) = -3\sin\left(\frac{1}{5}x\right)$



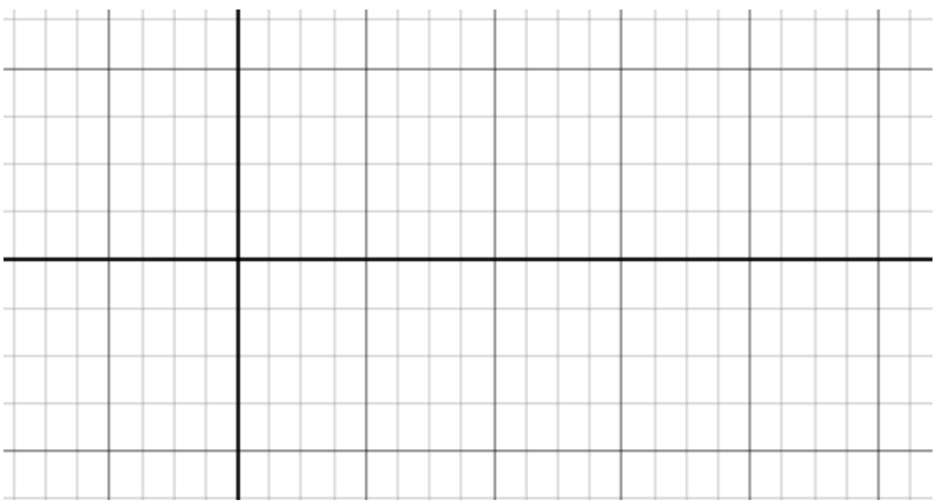
(Worksheet continued on next page)

(Worksheet continued)
This example will lead us into the second part of graphing sine and cosine where we put it all together.

(5) Graph $f(x) = 4\sin(2x)$



(6) Use the above graph to graph $g(x) = 4\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$

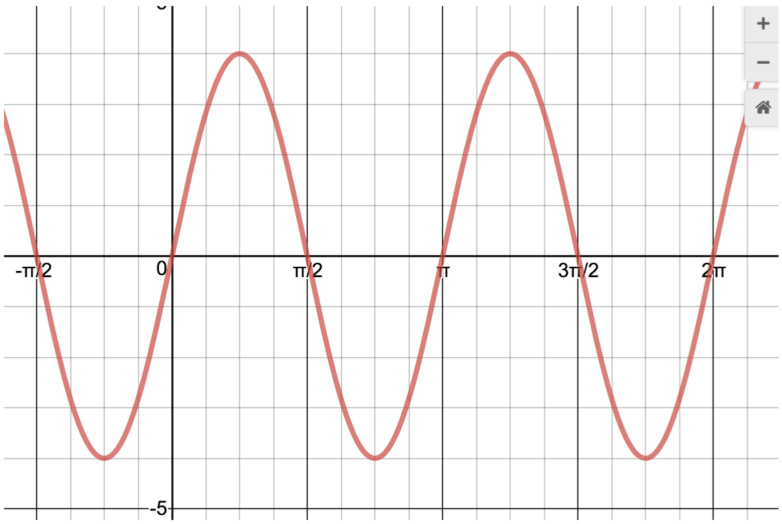


10.3ii Graphs of Sine and Cosine part ii

Combining change of period with horizontal shift

From homework:

Use the graph of $f(x)=4\sin(2x)$ to graph $g(x)=4\sin\left(2\left(x-\frac{\pi}{4}\right)\right)$



Notice, this function would normally be written $g(x)=$________________

But what was the horizontal shift?__________________________

Note: The horizontal shift is NOT _______________________

So given $g(x)=$____________________ to find the horizontal shift, we either have to factor out the 2

or divide ________________ by 2

Theorem 10.6. For $\omega > 0$, the graphs of

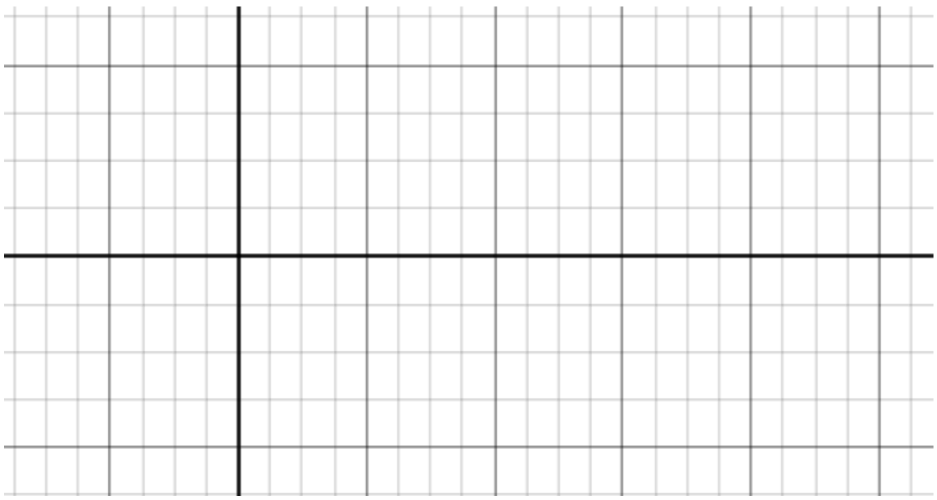
$$S(t) = A\sin(\omega t + \phi) + B \quad \text{and} \quad C(t) = A\cos(\omega t + \phi) + B$$

- have period $T = \frac{2\pi}{\omega}$
- have phase shift $-\frac{\phi}{\omega}$
- have amplitude $|A|$
- have vertical shift or 'baseline' B

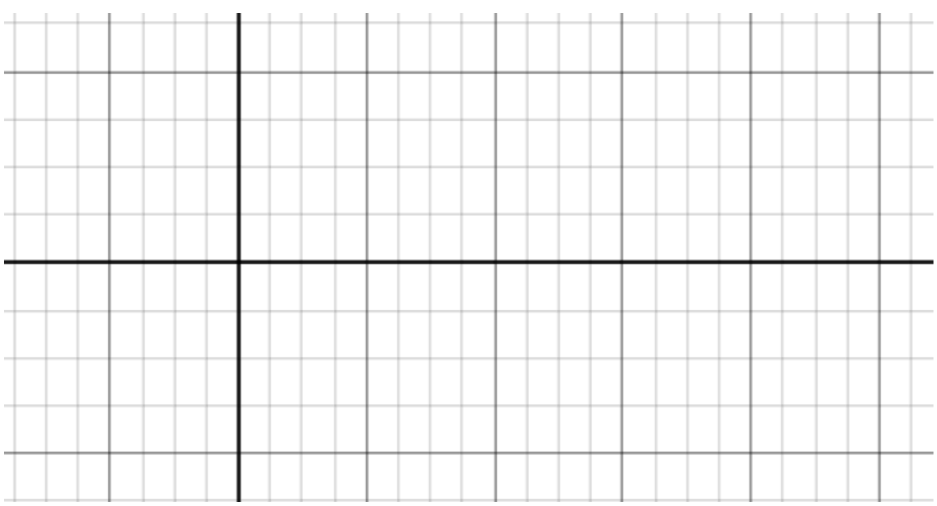
(I prefer the idea of factoring out ω so $S(t)=A\sin\left(\omega\left(t+\frac{\phi}{\omega}\right)\right)$ which shows a shift of $\frac{\phi}{\omega}$ left.

Examples

(1) Graph $f(x) = 2\sin\left(3x - \frac{\pi}{4}\right)$



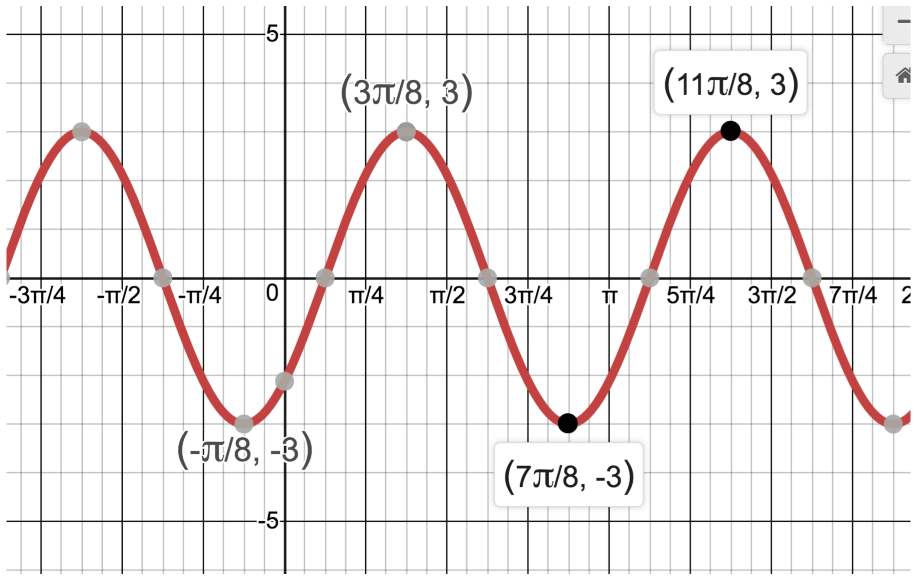
(2) Graph $f(x) = 3\cos\left(\pi x + \frac{\pi}{6}\right)$



How would we graph $f(x) = -3\cos\left(\pi x + \frac{\pi}{6}\right) + 1$?______________________________________

Using a graph to find the equation.

Often, we are provided with observational data and we wish to find an equations to model the physical situation. Fin equation corresponding the graph below. For one of the labeled points, check that it satisfies your equation.



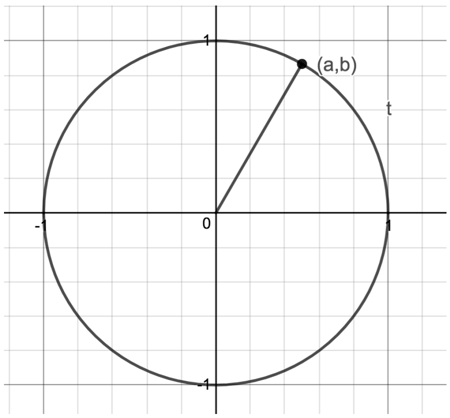
Measure the amplitude, half the distance from the lowest point to the highest. This is A.

Measure the period. Use this to get ω since 2π/ω is the period.

Now to finish, we need to find the ϕ of $y = A\sin(\omega x + \phi)$ or $y = A\cos(\omega x + \phi)$. To do this, think of the factored form $y = A\sin\left(\omega\left(x + \frac{\phi}{\omega}\right)\right)$ or $y = A\cos\left(\omega\left(x + \frac{\phi}{\omega}\right)\right)$. Read the shift from the graph. There are many possible answers depending on whether you are picturing it as a shift of the cosine graph or the sine graph. Put the shift into the factored form of the equation.

Ex: See example in book, page 854

B2 Trigonometric Functions of Acute Angles: "Right Triangle Trigonometry"

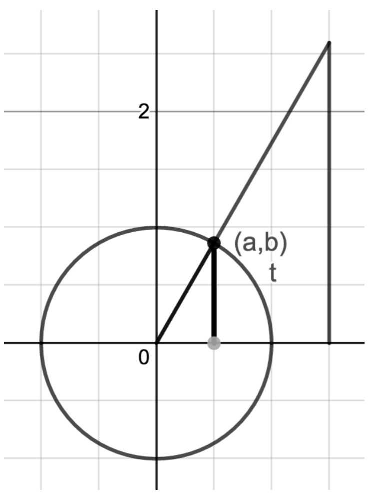


$\sin(t) = \sin(\theta) = \quad = \underline{\quad\quad} = \quad \underline{\quad\quad\quad}$

$\cos(t) = \cos(\theta) = \quad = \underline{\quad\quad} = \quad \underline{\quad\quad\quad}$

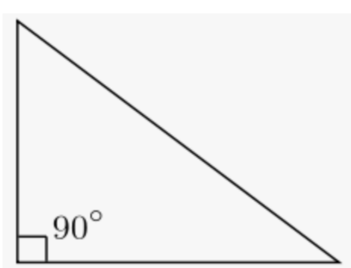
Because of the properties of similar triangles, these ratios still apply even if your triangle in not in the unit circle.

Example: (3-4-5)

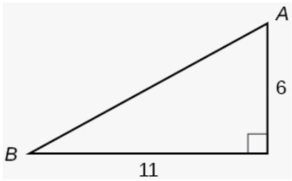


These definitions are consistent with the unit circle definitions, but are can be more useful in application problems.

The Other Ratios: Additional Trigonometric Functions:



For the angle θ below, find the value of the six trigonometric functions of θ.



$\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

In the above triangle, let α be the angle other acute angle (so α and θ are complementary). Find

$\sin(\alpha) =$ $\csc(\alpha) =$

$\cos(\alpha) =$ $\sec(\alpha) =$

$\tan(\alpha) =$ $\cot(\alpha) =$

Notice the relationship in the trig values of complementary angles.

$\sin(\alpha) =$ ________________

$\sec(\alpha) =$ ________________

$\tan(\alpha) =$ ________________

Identities

relationships hold for all acute angles.

41. $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ 42. $\csc(\theta) = \dfrac{1}{\sin(\theta)}$ 43. $\sec(\theta) = \dfrac{1}{\cos(\theta)}$

For Exercises 44 - 46, it may be helpful to recall that $90^\circ - \theta$ is the measure of the 'other' acute angle in the right triangle besides θ.

44. $\cos(\theta) = \sin(90^\circ - \theta)$ 45. $\csc(\theta) = \sec(90^\circ - \theta)$ 46. $\cot(\theta) = \tan(90^\circ - \theta)$

For Exercises 47 - 49, it may be helpful to remember that $a^2 + b^2 = c^2$:

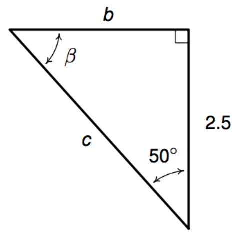
47. $(\cos(\theta))^2 + (\sin(\theta))^2 = 1$ 48. $1 + (\tan(\theta))^2 = (\sec(\theta))^2$ 49. $1 + (\cot(\theta))^2 = (\csc(\theta))^2$

The last identities are called the Pythagorean Identities and will discussed more in 11.1

Note: $\sin^2(\theta)$ is used to mean $(\sin(\theta))^2$, that is $(\sin(\theta))(\sin(\theta))$

The right triangle trigonometric formulas can be used to find missing parts of a right triangle:

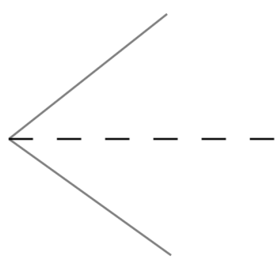
4. Find β, *b*, and *c*.



Calculator Usage: Exact vs. Approximate and using calculator storage for best approximation

<u>Application Examples</u>

Angle of elevation or inclination



Angle of depression

Note: these are always measured from the horizontal

34. A guy wire 1000 feet long is attached to the top of a tower. When pulled taut it makes a 43° angle with the ground. How tall is the tower? How far away from the base of the tower does the wire hit the ground?

33. From the observation deck of the lighthouse at Sasquatch Point 50 feet above the surface of Lake Ippizuti, a lifeguard spots a boat out on the lake sailing directly toward the lighthouse. The first sighting had an angle of depression of 8.2° and the second sighting had an angle of depression of 25.9°. How far had the boat traveled between the sightings?

Using known trig values to find others:

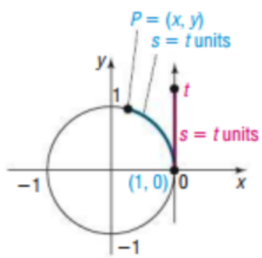
Given a trig function value for an acute angle θ, we can find the values of the other trigonometric functions for that same angle. (We will revisit this type of problem in 10.4 with θ not necessarily acute.)

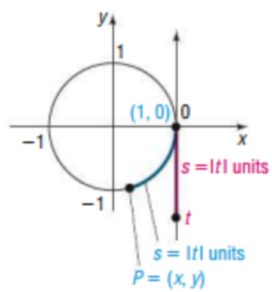
Ex: Given that θ is an acute angle and that $\sin(\theta) = \frac{1}{4}$, find the values of the other trig. Functions

10.4i   Unit Circle Definitions of the Other Trig Functions

Back to the Unit Circle:
Extending the definitions of the additional 4 trigonometric functions to the functions of a real number (Unit Circle Definitions).

Recall:
Consider the real number line corresponding values of t aligned next to the unit circle as shown. If this number line were wrapped around the unit circle, then every number t would correspond to a point P(x,y) on the unit circle $x^2 + y^2 = 1$.





$\sin(t) = \sin(\theta) =$ $\csc(t) = \csc(\theta) =$

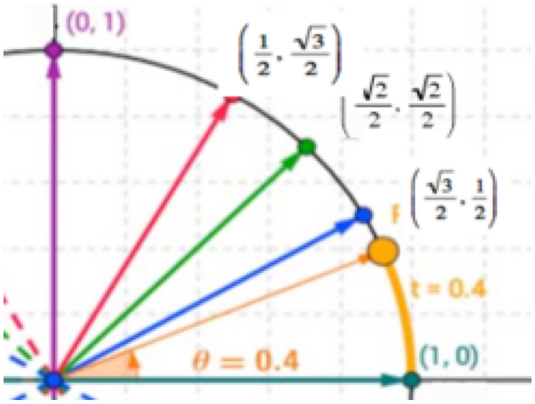
$\cos(t) = \cos(\theta) =$ $\sec(t) = \sec(\theta) =$

$\tan(t) = \tan(\theta) =$ $\cot(t) = \cot(\theta) =$

What are the domains for the 4 new trig. functions?

Signs of Trig functions in each quadrant:

Trig values of key angles revisited:



$\tan\left(\frac{\pi}{2}\right) =$ ____________________

$\tan\left(\frac{\pi}{3}\right) =$ __________________

$\tan\left(\frac{\pi}{4}\right) =$____________________

$\tan\left(\frac{\pi}{6}\right) =$ _____________________

$\tan(0) =$ _______________________

You should memorize the above tangent values: We will use them to find others.

Ex: Find $\tan\left(\frac{5\pi}{6}\right)$

Thought process for finding $\tan\left(\frac{5\pi}{6}\right)$

Locate $\frac{5\pi}{6}$. *It is a* _______________ *"type" angle in Q* ____________

What is the tangent of the reference angle ____________ *:* ________________

Attach a negative sign if needed based on what quadrant terminal side of $\frac{5\pi}{6}$ *resides in.*

In Q _________________ *tangent is* _________________ *so* $\tan\left(\frac{5\pi}{6}\right)$

Ex: Find $\tan\left(225^{\circ}\right)$

Time to practice: Find the following trig values exactly

Find each of the following

(a) cos (315°) = ______________

(b)) sec (π/4) = ____________________

(c) tan (330°) = _______________

(d) cot (- π/2)= ____________________

(e) tan (90°) = _______________

(f) tan (4π/3)= ____________________

(g) csc (390°) = ______________

(h) cos (7π/6) = ____________________

Solving Trigonometric Equations revisited:

Solve: $\tan(t) = 1$

This is saying, find the real number (arc length or corresponding angle, in radians) whose corresponding point on the unit circle has y/x value of 1. Note: Unless you know the tangent values of the key angles directly, this can be challenging.

How many terminal sides are there corresponding to this__________________________

How many values of t? (or think in angles) ______________________________

How do we express infinitely many answers?______________________________

Note: we can write this in a more compact way: ___________________________

Sometimes we are asked to solve for t on a restricted domain:

Solve: $\tan(t) = 1$ for $0 < t < 2\pi$ ____________________________

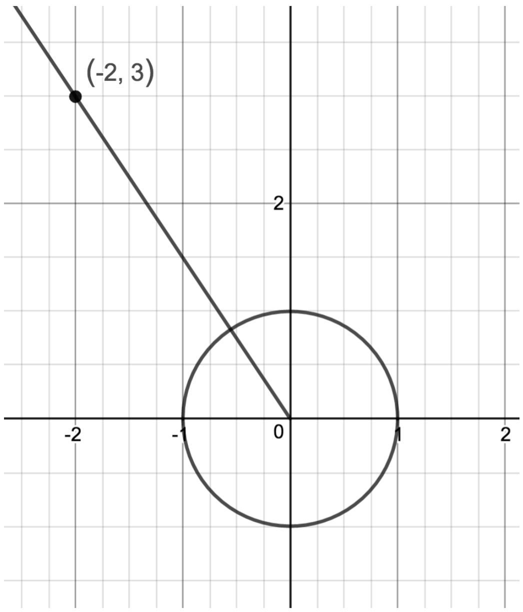
Solve: $\tan(t) = 1$ for $-\frac{\pi}{2} < t < \frac{\pi}{2}$ ____________________________

Example: Find all solutions:

$\tan(t) = -\sqrt{3}$ $\csc(t) = 2$ $\cot(t) = \sqrt{3}$

10.4ii Trigonometric Values of Angles Beyond the Unit Circle

Suppose we do not have a point on the terminal side of an angle where it intersects the unit circle, nor do we have an acute angle where we can use the right triangle definitions, how can we extend the trig. definitions to any angle, if we know any point of the terminal side.



In general,

Theorem 10.9. Suppose $Q(x, y)$ is the point on the terminal side of an angle θ (plotted in standard position) which lies on the circle of radius r, $x^2 + y^2 = r^2$. Then:

- $\sin(\theta) = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$
- $\cos(\theta) = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$
- $\tan(\theta) = \frac{y}{x}$, provided $x \neq 0$.
- $\sec(\theta) = \frac{r}{x} = \frac{\sqrt{x^2 + y^2}}{x}$, provided $x \neq 0$.
- $\csc(\theta) = \frac{r}{y} = \frac{\sqrt{x^2 + y^2}}{y}$, provided $y \neq 0$.
- $\cot(\theta) = \frac{x}{y}$, provided $y \neq 0$.

Example (text pg 836)

1. Suppose that the terminal side of an angle θ, when plotted in standard position, contains the point $Q(4, -2)$. Find $\sin(\theta)$ and $\cos(\theta)$.

2. Suppose $\frac{\pi}{2} < \theta < \pi$ with $\sin(\theta) = \frac{8}{17}$. Find $\cos(\theta)$.

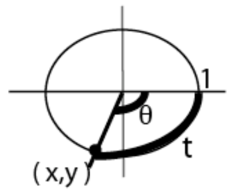
Summary of the Trig Definitions

THE DEFINITIONS OF THE TRIGONOMETRIC FUNCTIONS:

The definitions of the trigonometric functions are given in three different ways, depending on the situation.

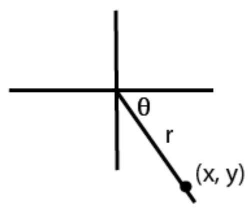
(1) The most general definition allows us to discuss the trig functions as being functions of a ***real number***, not just an angle. Given any real number t, let the point P(x,y) be a corresponding point on the **unit** circle determined by moving a distance of $|t|$ units around the circle starting at the point (1,0) and moving in the counter clockwise direction if t > 0, clockwise if t < 0. The central angle θ corresponding to the real number input t would be an angle of t radians. In this case,

$$\sin t = \sin\theta = y$$
$$\cos t = \cos\theta = x$$
$$\tan t = \tan\theta = \frac{y}{x}$$



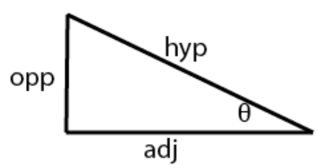
(2) In the case where the point P(x,y) is any point the terminal side of an angle θ in standard position, not necessarily on the unit circle, and **r** is the distance from P to the origin, then

$$\sin\theta = \frac{y}{r}$$
$$\cos\theta = \frac{x}{r}$$
$$\tan\theta = \frac{y}{x}$$



(3) In the special case where θ is an acute angle in a right triangle, the following definitions may be used.

$$\sin\theta = \frac{opp}{hyp}$$
$$\cos\theta = \frac{adj}{hyp}$$
$$\tan\theta = \frac{opp}{adj}$$

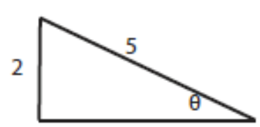


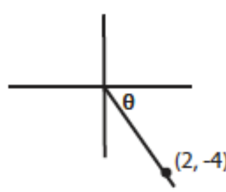
FINDING THE VALUES OF THE TRIG FUNCTIONS

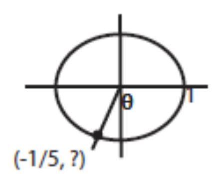
Finding the values of trig functions depends on what information is given.

(1) Given lengths of sides of a right triangle or a point on the terminal side of an angle or a point on the unit circle, we use the appropriate definition.

<u>example</u>: Given the following figures, find:







(a) cos θ=___________________
(b) csc θ=___________________

(c) tan θ=_____________
(d) sin θ=_____________

(e) sin θ=____________
(f) cot θ=____________

11.1 The Pythagorean Identities / Intro to Proving Identities

Previously, we considered the identities:

Theorem 11.1. Reciprocal and Quotient Identities: The following relationships hold for all angles θ provided each side of each equation is defined.

- $\sec(\theta) = \dfrac{1}{\cos(\theta)}$
- $\cos(\theta) = \dfrac{1}{\sec(\theta)}$
- $\csc(\theta) = \dfrac{1}{\sin(\theta)}$
- $\sin(\theta) = \dfrac{1}{\csc(\theta)}$
- $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$
- $\cot(\theta) = \dfrac{1}{\tan(\theta)}$
- $\tan(\theta) = \dfrac{1}{\cot(\theta)}$

Just as we needed to be good at manipulating algebraic expressions, we need to become proficient at manipulating trigonometric functions. Identities are powerful tools we will use to do this. You will need to memorize these (and future) identities.

Use the above identities to:

Find the following trigonometric function value: If $\sin(\theta) = \frac{-2}{3}$ find $\csc(\theta)$

Simplify the expression: $\left(\sec(\theta) + \tan(\theta)\right)\left(\cos(\theta)\right)$

Solve the equation: $\tan(t)\cos(t) = \frac{1}{2}$

Theorem 11.3. The Pythagorean Identities:

1. $\cos^2(\theta) + \sin^2(\theta) = 1$.

   Common Alternate Forms:

   - $1 - \sin^2(\theta) = \cos^2(\theta)$
   - $1 - \cos^2(\theta) = \sin^2(\theta)$

2. $1 + \tan^2(\theta) = \sec^2(\theta)$, provided $\cos(\theta) \neq 0$.

   Common Alternate Forms:

   - $\sec^2(\theta) - \tan^2(\theta) = 1$
   - $\sec^2(\theta) - 1 = \tan^2(\theta)$

3. $1 + \cot^2(\theta) = \csc^2(\theta)$, provided $\sin(\theta) \neq 0$.

   Common Alternate Forms:

   - $\csc^2(\theta) - \cot^2(\theta) = 1$
   - $\csc^2(\theta) - 1 = \cot^2(\theta)$

Note: $\sin^2(\theta)$ is used to mean $(\sin(\theta))^2$, that is $(\sin(\theta))(\sin(\theta))$

Derivation of the Pythagorean Identities (we can use any of the 3 trig definitions to derive these)

Use the above identities to:

Finding Trig Values Using a given Trig Value

Find the following trigonometric function value:
(note: we have many ways to do this type of problem (see page 28 and 32) **Also see math 8 page** Three-Ways to Find Trig Values When Given One Value Handout)

(1) If θ is a quadrant 2 angle with , $\sin(\theta)=\frac{2}{3}$ find $\cos(\theta)$

(2) If $\tan(t)=\frac{-1}{4}$ and $\sin(t)<0$ find $\cos(t)$

Simplifying Expressions

Simplify the expression:

(1) $1+3\cos^2(\theta)-\sin^2(\theta)$

(2)

$$\frac{2}{1-\sin(x)}-\frac{2}{1+\sin(x)}$$

Proving Identities.

Prove the identity:

$$\frac{\cos(t)}{\sin^2(t)} = \csc(t)\cot(t)$$

<u>Presentation in proofs is very important</u>. You are trying to convince the ***reader*** *that the statement is true.*
The goal is to start with one side of the equation and connect it to the other using clear simplifications that could be followed by any average Trig. student. (alternately you can work on each side and meet in the middle)

1) Start by rewriting the original form of the side you are beginning with, without making any simplifications to it.

2) Do not write "=" until you have shown "="

3) Do not treat as an equation, performing operations to both sides.

4) Draw conclusion to show you have finished.

(Text has many good examples pg 908)

Prove the identity:

(1)

$$\frac{\cos(t)}{1-\tan(t)} + \frac{\sin(t)}{1-\cot(t)} = \sin(t)+\cos(t)$$

(2)

$$\frac{\sin(x)}{1-\cos(x)} = \frac{1+\cos(x)}{\sin(x)}$$

MATH-8 TEST Unit 2
SAMPLE

100 points **NAME:**___________________

This test is in two parts. On part one, you may not use a calculator; on part two, a (non-graphing) calculator is necessary. When you complete part one, you turn it in and get part two. Once you have turned in part one, you may not go back to it. You will show all work on the test paper, no scratch paper is allowed.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following: (2 points each)

(a) cos (315°) = _____________ (b) sin (π) = _____________________

(c) tan (330°) =______________ (d) cot (- π/2)= ___________________

(e) tan (90°) = ______________ (f) sec (π/4) = ____________________

(g) csc (390°) = _____________ (h) cos (7π/6) = ___________________

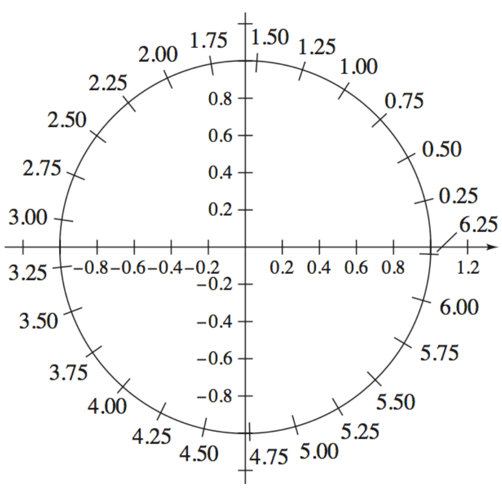
(i) sin (-150°) =______________ (j) tan (- π/6)= __________________

(2) Use the figure to (4 points)

(a) approximate the value of sin 5 ___________ cos 2 __________

(b) find a value of t such that cost ≈ -0.8 ____________

(c) find a value of t such that sint ≈ 0.4 ____________



NAME: ____________________

MATH 8 Sample Test 2

PART TWO - CALCULATORS ALLOWED (non-graphing)

Show your work on this paper. EXACT answers are expected unless otherwise specified. Show scales on graphs and label highs and lows. Give units in answers when appropriate.

Fill in the blanks. (2 points each)

(1) f(t) = cost Is even, odd, or neither ______________

(2) What is the amplitude of $f(t) = -\frac{1}{2}\sin(3t+\pi) - 4$?____________________

(3) If the point (-3, 7) is on the terminal side of θ, find sinθ________________

(4) In which quadrant, if any, is tanθ < 0 AND sinθ > 0 (both true) __________________

(5) The domain of f(t)=tan(t) is __________________________________

(6) Using your calculator, find approximations for the following, correct to 3 decimal places. (1 point each)

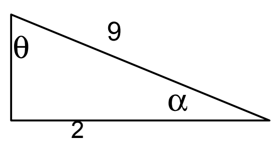
(a) sec 39° ≈ ______________

(b) $\tan(-3\pi/8)$ ≈ _________________

(c) $\frac{4}{\tan 12^\circ + 7}$ ≈ ___________

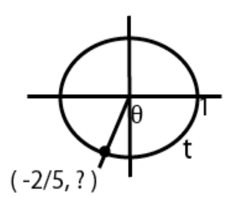
(d) cos 4 ≈ _____________________

(7) Given the following right triangle, find sinα, , cscθ, tanθ. (1 point each)



sinα= _______ cscθ=__________ tanθ=______.

(8) Given the unit circle below with the coordinates of P$\left(-\frac{2}{5}, ?\right)$, find sinθ , tant. (2 point each)



sinθ= ____________ tant = __________

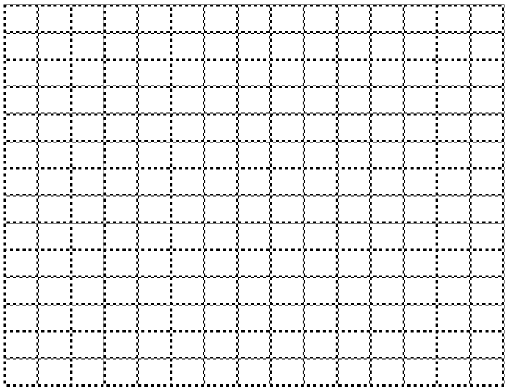
(9) Given cos θ = $\frac{-5}{13}$ and θ is in Quadrant II, find: (2 points each)

(a) sin θ= _________________

(b) sec θ ________________________

(10) Sketch the following graphs. (clearly show scale, graph at least one period, label coordinates of highs and lows) (4 points)

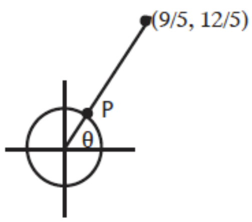
$g(x) = -2\cos(3x)$



(11) Given $\sec\theta = 3$ and $\tan\theta < 0$ find: (2 points each)

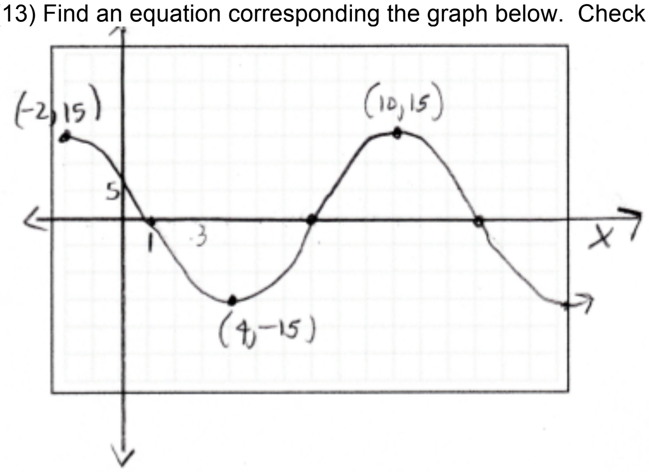
(a) $\sin\theta =$ ________________ (b) $\cot\theta$ ________________________

(12) Given the figure below, with point P on the unit circle, find (2 points each)



(a) $\cos\theta$ =____________ (b) $\tan\theta$ = _________ (c) coordinates of point P_____________

(13) Find an equation corresponding the graph below. Check a point. (4 points)



(14) A person sitting at the top of a 200 foot cliff at the edge of the ocean observes a ship directly offshore. The angle of depression from the person to the ship is 23 degrees. How far is the ship from shore (exact and approximate). (3 points)

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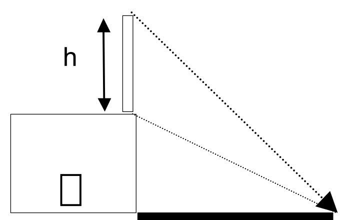
(15) At a point on the ground 200 feet from the base of a building, the angle of elevation to the bottom of a smokestack on the top of the building is 35°, and the angle of elevation to the top of the smokestack is 53°. Find the height, h, of the smokestack exactly.

(5 points)



(16) Solve the following trig equations. If not restriction is given then find all solutions (2 pts each)

$\tan(t) = -1$ for $0 \le t < 2\pi$ ______________________

$\sec(x) = -2$ for $0 \le x < 2\pi$ ________________

$\cos(t) = \frac{\sqrt{3}}{2}$ __________________________

$\sin(t) = 0$ __________________

$\sin(t) = \frac{-\sqrt{2}}{2}$ for $\frac{-\pi}{2} \le t \le \frac{\pi}{2}$ _________________

$\tan(t) = \sqrt{3}$ for $0 \le t < 4\pi$ _________________

(17) Simplify: $\dfrac{\tan\theta + \cot\theta}{3\sec\theta\csc\theta}$ (simplifies to a number) (2 points)

(18) Prove the following Identity $1 - \dfrac{\sin^2\theta}{1+\cos\theta} = \cos\theta$ (5 points)

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(19) f(x)=4sin$\left(\frac{1}{2}x + \frac{\pi}{6}\right)$ (6 points)